Econ 802

Final Exam

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All questions have equal weight. If something is unclear, please ask.

1. Suppose an undergraduate student makes each of the following statements. Give the student a clear verbal response. Do <u>not</u> use any mathematics or graphs.

(a) "We never have increasing returns to scale in the real world. This would imply that firms become infinitely large, but real firms have finite production plans."

(b) "If the production function is a concave function of the input quantities, then the profit function is a concave function of the input prices."

(c) "The long run average cost curve must pass through the minimum point of each short run average cost curve."

2. The Acme Company has the production function $f(x_1, x_2) = [x_1^2 + x_2^2]^{1/2}$ where $x = (x_1, x_2) \geq 0$. The input prices are $w = (w_1, w_2) > 0$.

(a) Suppose Acme wants to minimize the cost of producing an output $y > 0$. Set up a Lagrangian function for this problem <u>without</u> using any Kuhn-Tucker multipliers. Solve the first order conditions for $x_1(w, y)$ and $x_2(w, y)$. Do you notice anything unusual about this solution? Explain your reasoning.

(b) Draw a graph for the isoquant associated with $y > 0$. Using graphical arguments, find the true conditional input demand functions $x_1(w, y)$ and $x_2(w, y)$. Be sure to describe what the firm would do for each possible $w > 0$. Explain your reasoning.

(c) Why was the solution in (a) incorrect? What is the smallest closed, convex, and monotonic input requirement set $V(y)$ that is consistent with the correct solution from (b)? Explain your reasoning using a graph.

3. Here are some questions about consumer theory.

(a) State the Slutsky equation for the effect of the price p_i on the Marshallian demand x_i for the same good. Identify the substitution and income effects. Then define a Giffen good and say what would be necessary in order for such a good to exist.

(b) Prove that the Slutsky equation from part (a) is true. Carefully explain each step in the proof and be explicit about any assumptions you are making.

(c) Write down the matrix version of the Slutsky equation with n goods, and indicate the dimensions of each vector and matrix. What empirical predictions arise from this equation? Why does consumer theory make these predictions?

4. There are two goods, x and y. There are n consumers having the identical utility function $u(x, y) = x(a - x) + y$. We impose $x \geq 0$ but ignore non-negativity for y. Each of the consumers has an identical endowment $w > 0$ of the y good and a zero endowment of the x good. There are also m firms with the identical long run cost function $c(x) = x(b + x)$ where $c(x)$ is the amount of the y good needed in order to produce the output level $x \geq 0$. We assume $a > b > 0$. The price of the x good is $p \geq 0$. The price of the y good is always equal to one.

(a) Give complete mathematical descriptions of the demand function for good x for an individual consumer and for the market as a whole. Show the market demand curve on a graph. Is there a finite upper bound on the aggregate quantity of the x good that consumers would be willing to buy in the market? Explain briefly.

(b) Give complete mathematical descriptions of the supply function for good x for an individual firm and for the market as a whole. Show the market supply curve on a graph. Can an individual firm ever have negative profit? Explain briefly.

(c) Solve for the equilibrium price p^* and the market quantity X^*. Then use graphs to show what happens to (p^*, X^*) when (i) n increases while m is held constant; (ii) m increases while n is held constant. Give a brief interpretation in each case.

5. Consider an economy with consumers $i = 1 \, . \, . \, n$ and goods $j = 1 \, . \, . \, k$. Each i has a utility function $u_i(x_i)$ where $x_i = (x_{i1} \, . \, . \, x_{ik}) \geq 0$ is i's consumption bundle. Each i also has an endowment $w_i = (w_{i1} \, . \, . \, w_{ik}) \geq 0$. Utility functions are differentiable, have positive first derivatives for all goods, and are strictly concave. The price vector is $p = (p_1 \, . \, . \, p_k) \geq 0$.

(a) First define the aggregate excess demand function $z(p)$. Then define a Walrasian equilibrium. Finally, show that if p^* is a WE price vector, then tp^* is also a WE price vector for any $t > 0$. Briefly interpret this result.

(b) State Walras's Law and prove that it is true. Then show that if the equilibrium prices have $p^* > 0$, the WE has $z_j(p^*) = 0$ for all j. Briefly interpret this result.

(c) A planner maximizes the social welfare function $\sum a_i u_i(x_i)$ where $(a_1 \, . \, . \, a_n) > 0$ are constants, subject to physical feasibility constraints on the aggregate supplies of the goods. The resulting allocation is x^*. Suppose you want to obtain the same allocation as a WE. What must be true about the equilibrium prices p^*? What must be true about the individual endowment vectors w_i for $i = 1 \, . \, . \, n$? Will the resulting WE necessarily be Pareto efficient? Explain your answers carefully.